FastTrack - MA109

Factoring Polynomials

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REEF Question

Hello! Just a fun way to take attendance today...

What is your favorite kind of ice cream?

Outline

1. The Greatest Common Factor
2. Common Binomial Factors and Factoring by Grouping
3. Factoring Quadratic Polynomials
4. Factoring Special Forms and Quadratic Forms
5. Practice

Section 1

The Greatest Common Factor

What is Factoring?

To **factor** an expression means to *rewrite the expression as an equivalent product*.

Example:

$2x^2 + 6x = 2(x^2 + 3x)$ 2 is a factor of $2x^2 + 6x$

The **greatest common factor** (or GCF) is the largest factor common to *all* terms in the polynomial.

Example:

$2x^2 + 6x = 2x(x + 3)$ 2x is the greatest common factor of $2x^2 + 6x$

Examples

Always factor out the GCF first!!

Factor each polynomial.

1. $12x^2 + 18xy - 30y$
   GCF $= 6$
   $12x^2 + 18xy - 30y = 6(2x^2 + 3xy - 5y)$
2. $x^5 + x^2$
   GCF $= x^2$
   $x^5 + x^2 = x^2(x^3 + 1)$

REEF Question

Find the GCF.

1. $-13n^2 - 52$

Examples

Factor out the GCF.

1. $-13n^2 - 52$
2. $9p^2 + 27p^3 - 18p^4$

Examples

Factor out the GCF.

1. $-13n^2 - 52$
   GCF= -13
   $-13n^2 - 52 = -13(n^2 + 4)$
2. $9p^2 + 27p^3 - 18p^4$
   GCF $= 9p^2$
   $9p^2 + 27p^3 - 18p^4 = 9p^2(1 + 3p - 2p^2)$

Section 2

Common Binomial Factors and Factoring by Grouping

Common Binomial Factors

If a polynomial has a common binomial factor, it can also be factored out using the distributive property.

Example:
$(x+3)x^2 + (x+3)5 = (x+3)(x^2+5)$

You try!

Factor: $x^2(x-2) - 3(x-2)$

Common Binomial Factors

If a polynomial has a common binomial factor, it can also be factored out using the distributive property.

Example:
$(x+3)x^2 + (x+3)5 = (x+3)(x^2+5)$

You try!

Factor: $x^2(x-2) - 3(x-2)$
$(x^2-3)(x-2)$

Factoring by Grouping

Factor $3t^3 + 15t^2 - 6t - 30$.
First factor out the GCF.

$$3t^3 + 15t^2 - 6t - 30 = 3(t^3 + 5t^2 - 2t - 10)$$

When we have a polynomial with 4 terms, this is our hint to factor by grouping.

$$\begin{aligned} 3t^3 + 15t^2 - 6t - 30 &= 3(t^3 + 5t^2 - 2t - 10) \\ &= 3([t^3 + 5t^2][-2t - 10]) \\ &= 3(t^2[t+5] - 2[t+5]) \\ &= 3((t+5)(t^2 - 2)) \end{aligned}$$

Examples

Factor by grouping.

1. $6h^3 - 9h^2 - 2h + 3$
2. $3x^2 - xy - 6x + 2y$

Examples

Factor by grouping.

1. $6h^3 - 9h^2 - 2h + 3$

$$\begin{aligned} 6h^3 - 9h^2 - 2h + 3 &= (6h^3 - 9h^2)(-2h + 3) \\ &= 3h^2(2h - 3) - 1(2h - 3) \\ &= (3h^2 - 1)(2h - 3) \end{aligned}$$

2. $3x^2 - xy - 6x + 2y$

$$\begin{aligned} 3x^2 - xy - 6x + 2y &= (3x^2 - xy)(-6x + 2y) \\ &= x(3x - y) - 2(3x - y) \\ &= (3x - y)(x - 2) \end{aligned}$$

Section 3

Factoring Quadratic Polynomials

Quadratic Polynomials

A quadratic polynomial is one that can be written as $ax^2 + bx + c$ where a, b, c are real numbers and $a \neq 0$. $ax^2 + bx + c$, where $a = 1$

When a=1, then x^2 can only be factored as $x \cdot x$. Consider the FOIL method.

$$\begin{aligned}(x+b)(x+a) &= x^2 + ax + bx + ab \\ &= x^2 + (a+b)x + ab\end{aligned}$$

Factoring Trinomials with a Leading Coefficient of 1

If the constant term is positive, the binomials will have *like* signs:

$$(x+ \quad)(x+ \quad) \text{ OR } (x- \quad)(x- \quad),$$

to math the sign of the linear (middle) term.
If the constant term is negative, the binomials will have *unlike*signs:

$$(x+ \quad)(x- \quad),$$

with the larger factor placed in the binomial whose sign matches the linear (middle) term.

Examples and REEF Question

Example:

$$\begin{aligned} -x^2 + 11x - 24 &= -1(x^2 - 11x + 24) \\ &= -1(x- \quad)(x- \quad) \\ &= -1(x - 8)(x - 3) \end{aligned}$$

You try!

Factor $x^2 - 10 - 3x$.

Examples

Example:

$$\begin{aligned} -x^2 + 11x - 24 &= -1(x^2 - 11x + 24) \\ &= -1(x- \quad)(x- \quad) \\ &= -1(x-8)(x-3) \end{aligned}$$

You try!

Factor $x^2 - 10 - 3x$.

$$\begin{aligned} x^2 - 10 - 3x &= x^2 - 3x - 10 \\ &= (x+ \quad)(x- \quad) \\ &= (x+2)(x-5) \end{aligned}$$

What happens when the coefficient of the leading term is not 1?
$ax^2 + bx + c$ where $a \neq 1$ Try the AC Method!

For factoring $Ax^2 + Bx + C$

1. Multiply $A \cdot C$.
2. List the factors of $A \cdot C$, and locate the two factors that add to give you B.
3. Add an "x" to your factors, and write them in place of Bx.
4. Factor by grouping.

Examples

Factor $6z^2 - 11z - 35$.

AC=(6)(-35)=-210

<u>Factors of 210</u>

$1 \cdot 210$

$2 \cdot 105$

$3 \cdot 70$

$5 \cdot 42$

$6 \cdot 35$

$7 \cdot 30$

$10 \cdot 21 \rightarrow 10 and - 21$

$14 \cdot 15$

$$\begin{aligned} 6z^2 - 11z - 35 &= 6z^2 + 10z - 21z - 35 \\ &= (6z^2 + 10z)(-21z - 35) \\ &= 2z(3z + 5) - 7(3z + 5) \\ &= (2z - 7)(3z + 5) \end{aligned}$$

Section 4

Factoring Special Forms and Quadratic Forms

Special Forms

Difference of Perfect Squares

$A^2 - B^2 = (A + B)(A - B)$

Perfect Square Trinomials

$A^2 + 2AB + B^2 = (A + B)^2$
$A^2 - 2AB + B^2 = (A - B)^2$

Note: The previous two special types can be factored using the AC method.

Sum or Difference of Two Perfect Cubes

$A^3 + B^3 = (A + B)(A^2 - AB + B^2)$
$A^3 - B^3 = (A - B)(A^2 + AB + B^2)$

Note: This type you should recognize as a special case, because you can't factor it using the AC method.

Examples

Factor these special cases.

1. $4s^2 - 25$

$$\begin{aligned} 4s^2 - 25 &= (2s)^2 - (5)^2 \\ &= (2s+5)(2s-5) \end{aligned}$$

2. $z^2 - 18z + 81$

$$\begin{aligned} z^2 - 18z + 81 &= (1z)^2 - 2(9)(1z) + (9)^2 \\ &= (z-9)^2 \end{aligned}$$

Section 5

Practice

Practice

Factor

1. $h^5 - 12h^4 - 3h + 36$

2. $50x^2 - 72$

3. $3p^2 - 13p - 10$

REEF Question

Factor

1. $3p^2 - 13p - 10$

Practice - SOLUTIONS

Factor

1. $h^5 - 12h^4 - 3h + 36$
   $(h - 12)(h^4 - 3)$
2. $50x^2 - 72$
   $2(5x - 6)(5x + 6)$
3. $3p^2 - 13p - 10$
   $(3p + 2)(p - 5)$